# Homework # 3

## Due 19 September 2007

1. A sample of methanol is compressed using a piston/cylinder apparatus loaded with kilogram masses. Relative volumes compared to a volume of 1.0000 at 20 °C and 1 kg $cm^{-2}$ are measured and recorded in the table. Using these data make an estimate of $\Delta\tilde{S}$ when methanol at 35 °C and 1 kg $cm^{-2}$ is compressed isothermally to 5000 kg $cm^{-2}$. The density of methanol at 20 °C and 1 kg $cm^{-2}$ is 0.7914 g $cm^{-3}$.

| Load (kg $cm^{-2}$) | 1 | 500 | 1000 | 2000 | 3000 | 4000 | 5000 |
|---|---|---|---|---|---|---|---|
| $V_{\text{rel}}$ (20 °C) | 1.0238 | 0.9823 | 0.9530 | 0.9087 | 0.8792 | 0.8551 | 0.8354 |
| $V_{\text{rel}}$ (50 °C) | 1.0610 | 1.0096 | 0.9763 | 0.9271 | 0.8947 | 0.8687 | 0.8476 |

2. Given the ordering $y^{(0)} = U = f(S, V, n_1, \ldots, n_C)$ express the following partial derivatives in a simpler form by playing the partial derivative game. Indicate if each term can be measured experimentally, and how you would measure it.

   (a) $\left(\frac{\partial y^{(2)}}{\partial V}\right)_{U_S, n_i}$

   (b) $\left(\frac{\partial U_{n_j}}{\partial U_S}\right)_{V, n_i}$

   (c) $\left(\frac{\partial y^{(1)}}{\partial V}\right)_{U_S, n_i}$

   (d) $\left(\frac{\partial S}{\partial U_S}\right)_{y^{(2)}, n_i}$

   The notation $J_M$ means $J_M = \left(\frac{\partial J}{\partial M}\right)$. E.g., $U_S = \left(\frac{\partial U}{\partial S}\right)_{V, n_i}$.

3. You have always been taught that for an ideal gas $\tilde{C}_v$ and $\tilde{C}_p$ are not independent, but are related through

$$\tilde{C}_p - \tilde{C}_v = R. \qquad (1)$$

   (a) Derive a general relationship between $\tilde{C}_v$ and $\tilde{C}_p$ for real fluids.

   (b) Prove Eq. (1) for an ideal gas.

   (c) Derive an expression for $\tilde{C}_p - \tilde{C}_v$ for a gas that obeys the truncated virial equation of state,

$$p = \frac{RT}{\tilde{V}} + RT\frac{B(T)}{\tilde{V}^2} \qquad (2)$$

   where $B(T)$ is a function of temperature only.

4. Evaluate the following expressions for a gas that obeys the truncated virial equation of state given by Eq. (2). You may assume that the constant volume heat capacity is given by $\tilde{C}_v = 5/2R$.

   (a) The Joule-Thompson coefficient: $\left(\frac{\partial T}{\partial p}\right)_{H,n}$

   (b) The speed of sound: $\nu_C = \left[\left(\frac{\partial p}{\partial \rho}\right)_{S,n}\right]^{1/2}$ where $\rho = m/V$, and $m$ is the mass.

   (c) $\left(\frac{\partial G}{\partial H}\right)_{T,n}$

   (d) $\left(\frac{\partial U}{\partial S}\right)_{T,n}$