Homework # 3

Due 30 September

1. You have always been taught that for an ideal gas $\tilde{C}_v$ and $\tilde{C}_p$ are not independent, but are related through

$$\tilde{C}_p - \tilde{C}_v = R. \quad (1)$$

(a) Derive a general relationship between $\tilde{C}_v$ and $\tilde{C}_p$ for real fluids.
(b) Prove Eq. (1) for an ideal gas.
(c) Derive an expression for $\tilde{C}_p - \tilde{C}_v$ for a gas that obeys the truncated virial equation of state,

$$p = \frac{RT}{\tilde{V}} + RT \frac{B(T)}{\tilde{V}^2} \quad (2)$$

where $B(T)$ is a function of temperature only.

2. Evaluate the following expressions for a gas that obeys the truncated virial equation of state given by Eq. (2). You may assume that the constant volume heat capacity is given by $\tilde{C}_v = \frac{5}{2}R$.

(a) The Joule-Thompson coefficient: $\left(\frac{\partial T}{\partial p}\right)_{H,n}$
(b) The speed of sound: $\nu_C = \left[\left(\frac{\partial p}{\partial \rho}\right)_{S,n}\right]^{1/2}$ where $\rho = m/V$, and $m$ is the mass.
(c) $\left(\frac{\partial G}{\partial T}\right)_{T,n}$
(d) $\left(\frac{\partial U}{\partial S}\right)_{T,n}$

3. Develop pressure explicit expressions for the residual functions (departure functions), $\Delta H'$ and $\Delta S'$ for the van der Waals equation of state:

$$p = \frac{RT}{\tilde{V} - b} - \frac{a}{\tilde{V}^2} \quad (3)$$

4. For this problem use the equation of state $p = \frac{RT}{\tilde{V}} + \frac{a}{\tilde{V}^2}$, where $a = 950 \text{ J m}^3/\text{mol}^2$, along with the ideal gas heat capacity $C'_v = a + bT + cT^2$ where $a = 32.8$, $b = 0.16$, and $c = 3.7 \times 10^{-4}$ and $C'_p$ is in units of $\text{J/(mol K)}$.

Compute the $\Delta U$, $\Delta H$, $\Delta S$ for a process that irreversibly compresses a fluid from $T_1 = 300 \text{ K}$, $P_1 = 1.01 \text{ bar}$ to $T_2 = 550 \text{ K}$, $P_2 = 61.7 \text{ bar}$.

5. A non-ideal gas with constant (ideal gas) heat capacity, $\tilde{C}'_v = 12.56 \text{ J/mol K}$ undergoes a reversible adiabatic expansion. The $pV T$ properties of the gas can be approximately described by the van der Waals equation of state given in Eq. (3), where $a = 0.1362 \text{ J m}^3/\text{mol}^2$, and $b = 3.215 \times 10^{-5} \text{ m}^3/\text{mol}$.

(a) Derive a set of equations to compute the change in internal energy for this system. Hint: Think carefully about what is held constant.
(b) Calculate the $\Delta U$ when the system is initially at $\tilde{V} = 2.5 \times 10^{-4} \text{ m}^3/\text{mol}$, $T = 494 \text{ K}$ and is expanded to twice the volume.